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A Relation Between Liquid Roll Moment and Liquid Side Moment

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Introduction

IN the summer of 1974, a 155-mm shell with a liquid payload was fired with a yawsonde telemetry unit. This projectile developed a large amplitude coning motion. When the coning motion exceeded forty deg, the telemetry record showed a very rapid despin of 40% in less than five s.[1] Since 1974, ten more observations of large coning motions and large despin rates of liquid-filled projectiles have been made.[2-4] In addition, 36 observations have been made of large coning motions and despin moments for projectiles with payloads of liquid-solid mixtures.[5] Indeed, all projectiles with liquid or liquid-solid payloads were observed to perform large coning motion and to experience large losses in spin.

This characteristic association of a large despin moment with a large coning motion for a projectile with a moving payload can become a diagnostic tool. Miller[6] suggested using this tool to determine the existence of small-amplitude, unstable, liquid-induced side moments. Spin measurements made during coning motion on a gyroscope predicted flight pitch instaiblities caused by very viscous liquids. These were observed in flight.[7]

The linear liquid-induced side moment was first computed by Stewartson[8] for an inviscid liquid payload by use of eigenfrequencies determined by the fineness ratio of the cylindrical container. Wedemeyer[9] introduced boundary layers on the walls of the container and was able to determine viscous corrections for Stewartson's eigenfrequencies, which could then be used in Stewartson's side moment calculation. Murphy[10] then completed the linear boundary layer theory by including all pressure and wall shear contributions to the liquid-induced side moment.

The first theoretical work on liquid-induced roll moments was done by Vaughn[11] in 1978. Although fair agreement was obtained with Miller's data, the work was marred by some hard-to-justify algebraic steps. Recently Vaughn et al.[12] developed an impressive numerical capability for computing roll moments of very high viscosity liquids and obtained excellent agreement with Miller's data. In Ref. 5, the linearized Navier-Stokes equations were used to develop a relationship between the liquid side moment and the liquid roll moment. This relationship can be used to predict the side moment from a measured roll moment. It is the purpose of this Note to give the results of this analysis and illustrate its predictions with yawsonde data recently analyzed by Pope.[13]

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Liquid Roll Moment

Two coordinate systems will be used in this Note: 1) the nonrolling aeroballistic $\tilde{X}\tilde{Y}\tilde{Z}$ system whose $\tilde{X}$-axis is fixed along the missile's axis of symmetry, and 2) the inertial XYZ system whose X-axis is tangent to the trajectory at time zero. Both coordinate systems have origins at the center of the cylindrical payload cavity, which is assumed to be at the center of mass of the projectile.

Although the general motion of a spinning projectile is the sum of two coning motions,[10] in this Note we will restrict ourselves to a single coning motion with amplitude α_c and phase angle ϕ_c. In terms of this coning motion, the liquid transverse moment and liquid roll moments must be odd and even functions of α_c, respectively, and are assumed to have the form

$$M_{L\tilde{Y}} + iM_{L\tilde{Z}} = m_L a^2 \dot{\phi}^2 \tau (C_{\mathrm{LSM}} + iC_{\mathrm{LIM}}) K_c e^{i\phi_c} \quad (1)$$

$$M_{L\tilde{X}} = m_L a^2 \dot{\phi}^2 [C_{\mathrm{LRM}_0} + \tau K_c^2 C_{\mathrm{LRM}}] \quad (2)$$

where m_L is the mass of liquid in a fully filled container, a is the maximum radius of the container, $\dot{\phi}$ is the spin rate, τ is the ratio of coning rate to spin rate, $\dot{\phi}_c/\dot{\phi}$, C_{LSM} is the liquid side moment coefficient, C_{LIM} is the liquid in-plane moment coefficient, and K_C is $\sin\alpha_c$.

When the liquid has reached a steady-state rolling and coning motion, the roll moment must vanish in the absence of coning motion. Thus C_{LRM} is the steady-state liquid roll moment coefficient due to coning motion while C_{LRM_0} is the

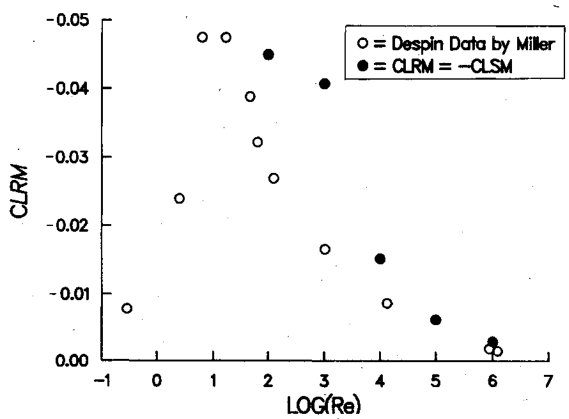

Fig. 1 C_{LRM} from gyroscope tests (Ref. 8), $c/a = 4.291$.

Table 1 Pope's results

Yawsonde	Re	τ	C_{LRM}	C_{SMEQ}	C_{SMA}
404[a]	2×10^6	0.090	−0.05	0.05	<0.01
1339	1.8×10^6	0.090	−0.02	0.04	−0.005
1866	45.2	0.123	−0.055	0.050	−0.004
1867	45.2	0.123	−0.055	0.050	−0.004
1868	45.2	0.123	−0.060	0.053	−0.004
1955	20	0.087	−0.04	0.04	−0.010
1293	10	0.090	−0.03	0.02	−0.008
1313	10	0.088	−0.02	0.025	−0.008
1585	10	0.091	−0.025	0.02	−0.008
1587	10	0.093	−0.04	0.02	−0.008
1588	10	0.095	−0.03	0.02	−0.008
1693[b]		0.094	−0.025	0.025	−0.005

[a]Fill ratio for 404 was 87%; all others were 100%. [b]This projectile contained white phosphorous-impregnated felt wedges.[14]

liquid roll moment due to transient liquid spinup. C_{LSM}, C_{LIM}, and C_{LRM} are even functions of α_c, and for linear fluid mechanics are independent of α_c. They are, of course, also functions of Reynolds number, τ, and container shape.

Since $\dot{\phi}^2\tau$ is $\dot{\phi}\dot{\phi}_c$, we are explicitly assigning equal weights to the spin rate and coning rate in the steady-state roll moment of Eq. (2). As the steady-state roll moment is a response to the coning motion, the appearance of $\dot{\phi}_c$ is natural. The primary characteristic of the high Reynolds number Stewartson-Wedemeyer theory is the existence of a dominant inertia wave mode. This resonance mode is caused by an interaction of the spinning liquid with the coning motion and thus the spin rate is as important as the coning rate. At very low Reynolds numbers, these inertia waves are strongly damped and the influence of spin on the liquid moment should be much weaker. Indeed, Miller[8] found no dependence of the roll moment on spin for $\tau<0.3$ and very low Reynolds numbers. If the roll moment is independent of spin, the liquid roll moment coefficient is proportional to τ ($C_{LRM}=\tau C_R$).

The frozen liquid value ($Re=0$) of the liquid in-plane moment coefficient for constant amplitude coning motion and a cylindrical container of length $2c$ is[12]

$$C_{LIM}=\tfrac{1}{2}-\tau[\tfrac{1}{4}+c^2/3a^2] \tag{3}$$

Thus the in-plane moment must always be strongly dependent on spin. The side-moment coefficient, however, for $\tau<0.3$ and low Reynolds number is probably proportional to τ ($C_{LSM}=\tau C_S$). For these conditions both C_S and C_R may be more appropriate nondimensional forms of these moments. (These new coefficients may, of course, be functions of τ.)

Roll/Side Moment Relation

In Ref. 5 it is shown that

$$C_{LRM}=-C_{LSM}+(\tau\epsilon/2)\,[1-(4/3)\,(c/a)^2],\ \text{or}$$

$$C_R=-C_S+(\epsilon/2)\,[1-(4/3)\,(c/a)^2] \tag{4}$$

where $\tau\epsilon=\dot{K}_c/K_c$.

This equation is valid for linear fluid mechanics, and states that for the simple case of constant amplitude motion ($\epsilon=0$) the liquid roll moment coefficient is the negative of the liquid side moment coefficient. Since no boundary layer assumptions were used to obtain Eq. (4), the equation should apply to the numerical results of Ref. 1. If the complete nonlinear fluid equations are used, additional significant contributions to the quadratic liquid roll moment may be present. It is interesting to note that for constant amplitude coning motion, Eq. (4) is the linear condition for the liquid moment about the trajectory to be zero.

$$M_{LX}=M_{L\tilde{X}}\cos\alpha_c+M_{LSM}\sin\alpha_c\doteq M_{L\tilde{X}}+M_{LSM}K_c=0 \tag{5}$$

where M_{LSM} is the liquid side moment determined by C_{LSM}.

Recently, Pope[13] very carefully analyzed the yawsonde records of twelve unstable liquid-filled projectiles. All projectiles showed a growing coning motion and a rapid despin. Two of these missiles carried low viscosity liquids ($Re\sim10^6$), four carried silicon oil ($Re\sim40$), five carried corn syrup ($Re\sim10$), and one had felt wedges impregnated with liquid white phosphorous.

Since the aerodynamic roll damping is easily measured, Pope was able to obtain liquid roll moment coefficients. He was unable to separate the aerodynamic and liquid side moments in the observed motion and, therefore, obtained "equivalent" side moment coefficients (C_{SMEQ}) containing both contributions. He was able to make estimates of the aerodynamic side moment coefficients (C_{SMA}). These three coefficients are given in Table 1 for the 12 yawsonde records. The rough agreement of C_{LRM} and C_{SMEQ} with Eq. (4) is very gratifying.

Finally, Miller's liquid roll moment data[6] for spinning cylinders coning at a constant cone angle of 20 deg are plotted in Fig. 1 as open circles. C_{LSM}'s are computed by the linear boundary layer theory of Ref. 10 and plotted as solid circles. The qualitative agreement for the boundary layer theory at such low Reynolds numbers is quite encouraging.

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